Parallel CLEAN: beyond the frequency domain

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The CLEAN algorithm\(^1,2\), despite its problematic convergence properties and its need for heuristic intervention and tuning, is the most commonly used means of attenuating sampling artifacts in images made through radio interferometry. Subsequent modifications by Conway\(^3\) and Sault and Wieringa\(^4\) adapted CLEAN to observations made using a wide fractional bandwidth, which are not well handled by the original algorithm. Here a generalized version of the Sault-Wieringa algorithm is applied to other cases where standard CLEAN also fails, including time-variable\(^1\) and off-pixel-centre sources. A 1-dimensional version of the latter technique can also be applied to extract non-integer frequency values from periodograms.

### Generalized parallel CLEAN: dirty image as a sum of convolutions.

<table>
<thead>
<tr>
<th>Dirty image (D)</th>
<th>(I_0)</th>
<th>(I_1)</th>
<th>(I_2)</th>
<th>(B_0)</th>
<th>(B_1)</th>
<th>(B_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Dirty image" /></td>
<td><img src="image2" alt="Convolution 1" /></td>
<td><img src="image3" alt="Convolution 2" /></td>
<td><img src="image4" alt="Convolution 3" /></td>
<td><img src="image5" alt="Beam 1" /></td>
<td><img src="image6" alt="Beam 2" /></td>
<td><img src="image7" alt="Beam 3" /></td>
</tr>
</tbody>
</table>

\(D = I_0 \ast B_0 + I_1 \ast B_1 + I_2 \ast B_2 + \ldots\)

### The parallel CLEAN algorithm\(^4\):

\[
R = D + B,
\]

\(A = B + B\)

Construct \(M\) such that \(m_x = A_x(0,0)\)

for \(x = 1, N_x\),

\[
R = \Sigma R_x \cdot (r_{max}),
\]

\[
f = \Sigma c_x / A_x(r_{max}) \quad (\text{v} \text{ually} < 1)
\]

\(c = M' c'\)

Save the vector clean component \(c\)

### Time-varying sources: the wide-band case

Expand the sky brightness map \(I(x,y,t)\) in basis functions:

\[
I(x,y,t) = \sum_{p=1}^{N_x} \sum_{q=1}^{N_y} A_{pq}(r) F_{pq}(v) T_{pq}(t)
\]

Each \(F_{pq} T_{pq}\) generates a ‘dirty beam’, giving \(N = N_x \times N_y\) beams in total.

Just as Högbom CLEAN models \(I(x)\) as sum of scalar clean components, so Sault-Wieringa CLEAN models \(A_{pq}(r)\) as a sum of vector clean components \(c'_{pq}\).

With a proper choice of time basis function, this expansion in the image plane allows one to avoid Gibbs ringing either at the boundaries or at gaps in the time sequence of the data.

Since light curves are expected to be more rapidly varying than spectra, \(N_y\) and therefore the total number of beams \(N\) may need to be large to obtain good modelling of the sources. But the algorithm (see top right) requires \(O(N^2)\) images to be stored. This can be reduced to \(O(N)\) if the beams are orthogonalized (eg via Gram-Schmidt) before commencing cleaning. Clean components can be recovered afterwards via a matrix inversion.

### Time-varying sources: narrow-band observation, unresolved source

If time-variation is restricted to a single point source in a narrow-band observation, perfect decomposition can be achieved with just 2 beams: the usual dirty beam for one, and a beam constructed from the entire light curve of the data for the other. Each pixel of the dirty image must be a combination of these two.

### Sub-pixel cleaning (1D example)

The ‘beams’:

\[
B_0 = \delta[S(x)2]
\]

\[
B_1 = \delta[\pi x/S(x)]
\]

Fourier transforming gives \(c = \delta[S(x+\varphi)] = \delta[B_0 \ast B_1]\)

\(A, f\) and \(\varphi\) are easily recovered after Sault-Wieringa processing.

Higher orders ➔ greater accuracy.

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